Nonlinear Coherent Perfect Absorbing Wavefronts (NLCPA) on Graphs

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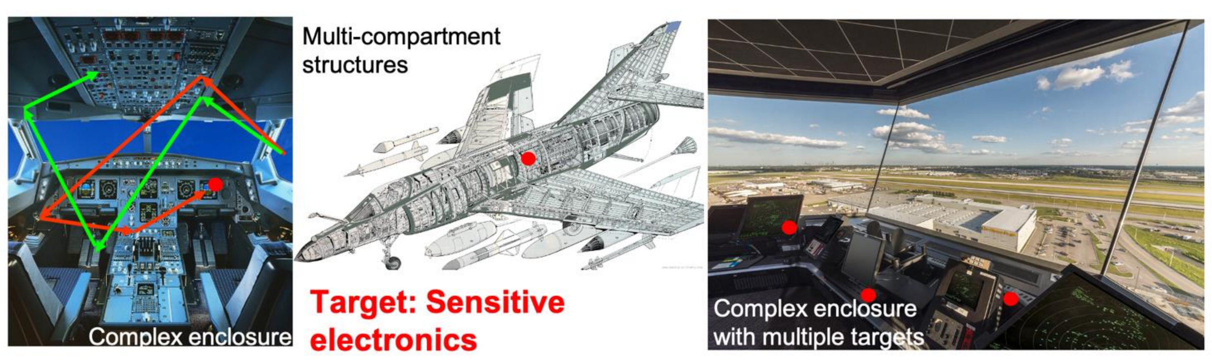
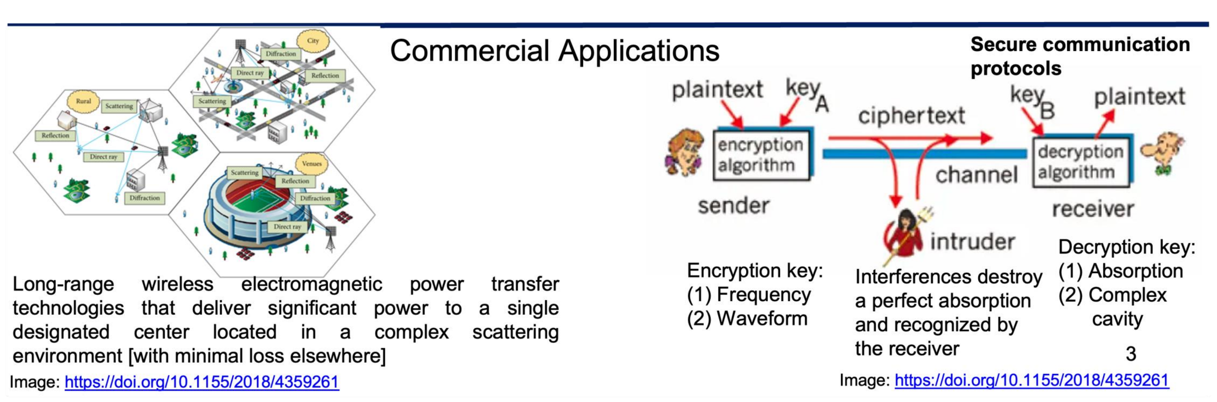
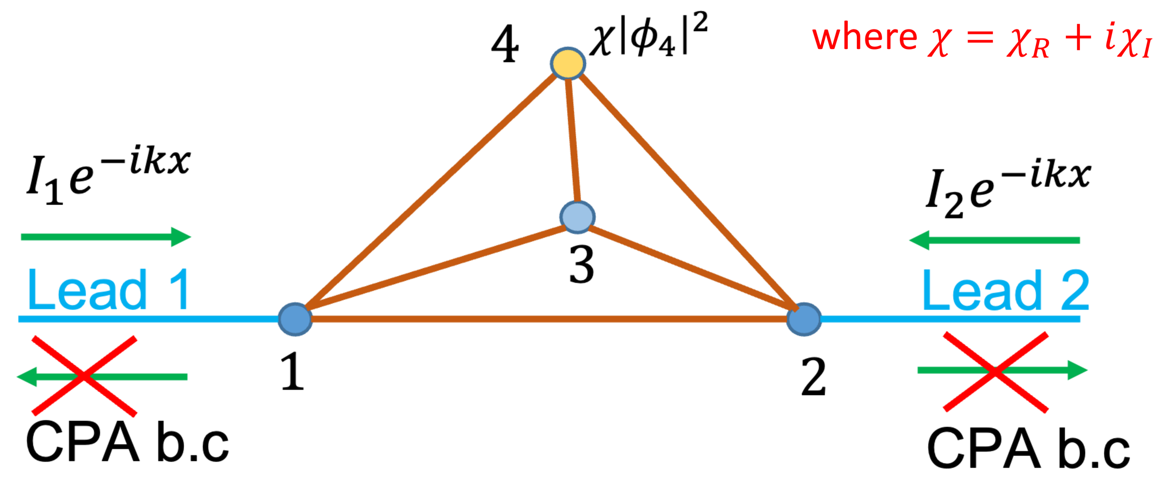
Abstract

We develop the theoretical framework necessary for the design of agile electromagnetic waveforms which under non-linear scattering conditions can target sensitive electronic elements embedded inside complex (reverberate) enclosures. Our computational schemes are tested against experimental realities with microwave settings.





1. Nonlinear Coherent Perfect Absorbing Wavefronts (NLCPA) on Graphs



- Wave equation

$$\frac{d^2}{dx^2}\psi_{\mu\beta}(x) + \frac{\omega^2\epsilon_0}{c^2}\left[(\lambda_\mu + \chi|\phi_N|^2)\delta(x) + 1\right]\psi_{\mu\beta} = 0$$

k=$\sqrt{\epsilon_0}\omega/c$ ω =2πν

- Continuity of wave function at each vertex

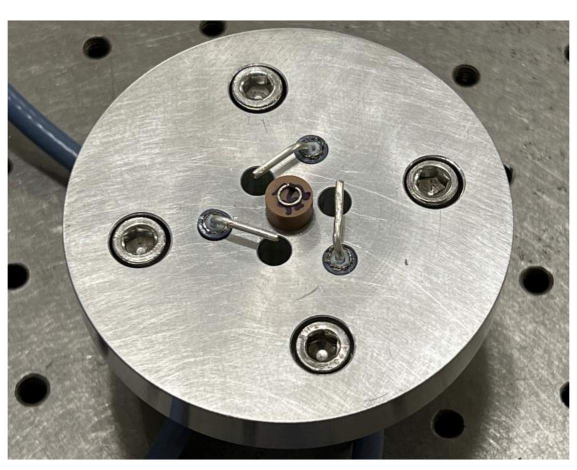
$$\psi_{\mu\beta}(x_{\mu\beta} = 0) = \phi_\mu, \psi_\mu(x=0) = \phi_\mu$$

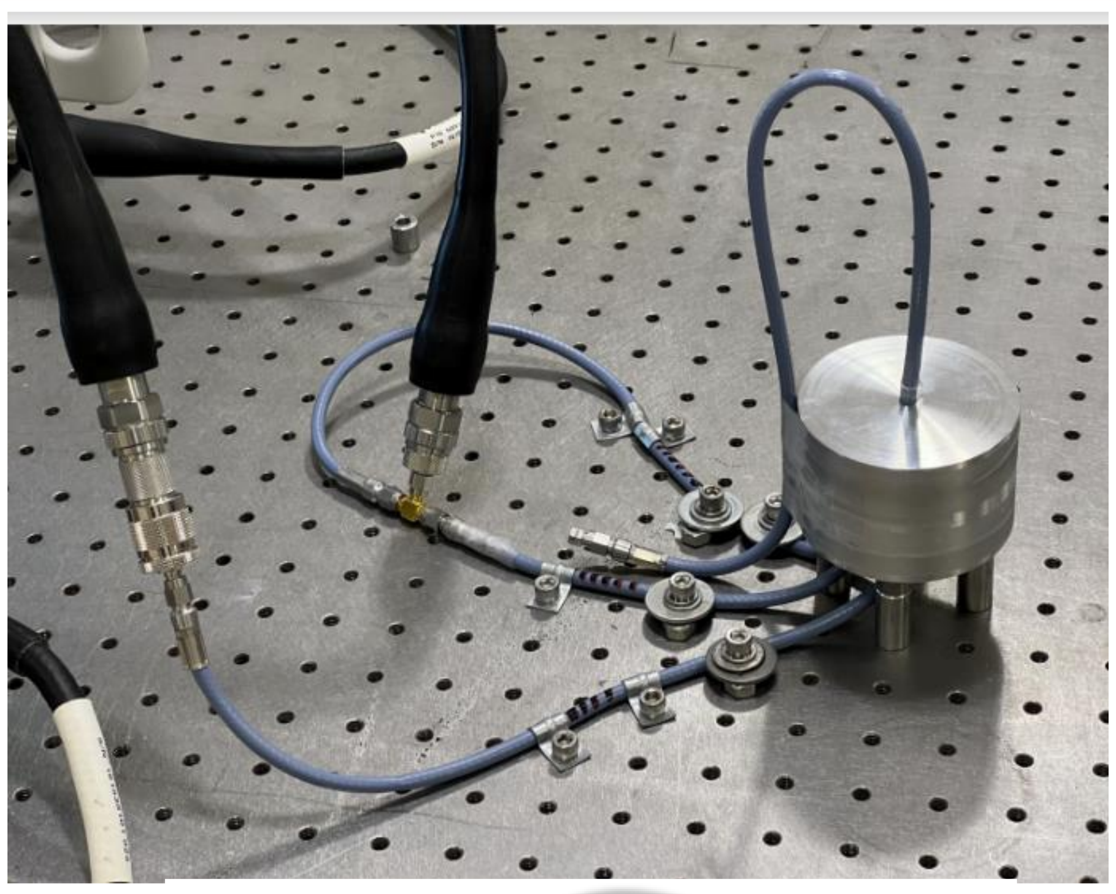
$\psi_{\mu\beta}$ is wave function on bond $\mu - \beta$,
ψ_μ is wave function on μ −lead,
ϕ_μ is wave on μ −vertex

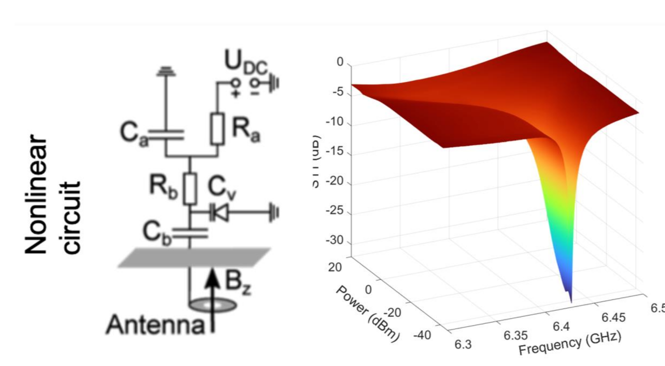
- Current conservation at each vertex

$$\sum_\beta \left.\frac{d\psi_{\mu\beta}}{dx_{\mu\beta}}\right|_{x_{\mu\beta}=0} + (\delta_{\mu,1}+\delta_{\mu,2})\left.\frac{d\psi_\mu}{dx}\right|_{x=0} = -k^2(\lambda_\mu + \delta_{\mu,N}\chi|\phi_N|^2)\phi_\mu$$

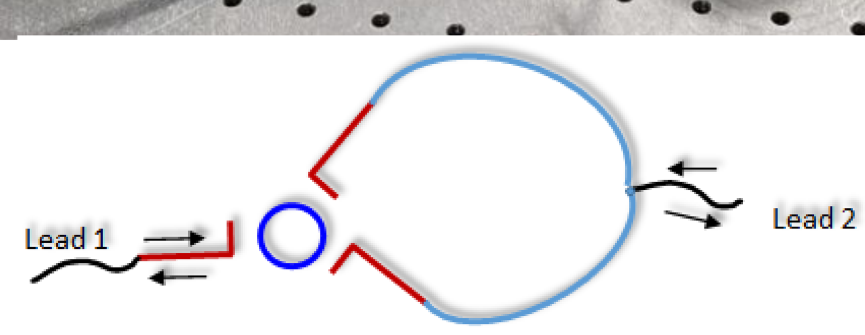
3. NLCPA in Ring Graphs: Theory vs. Experiment

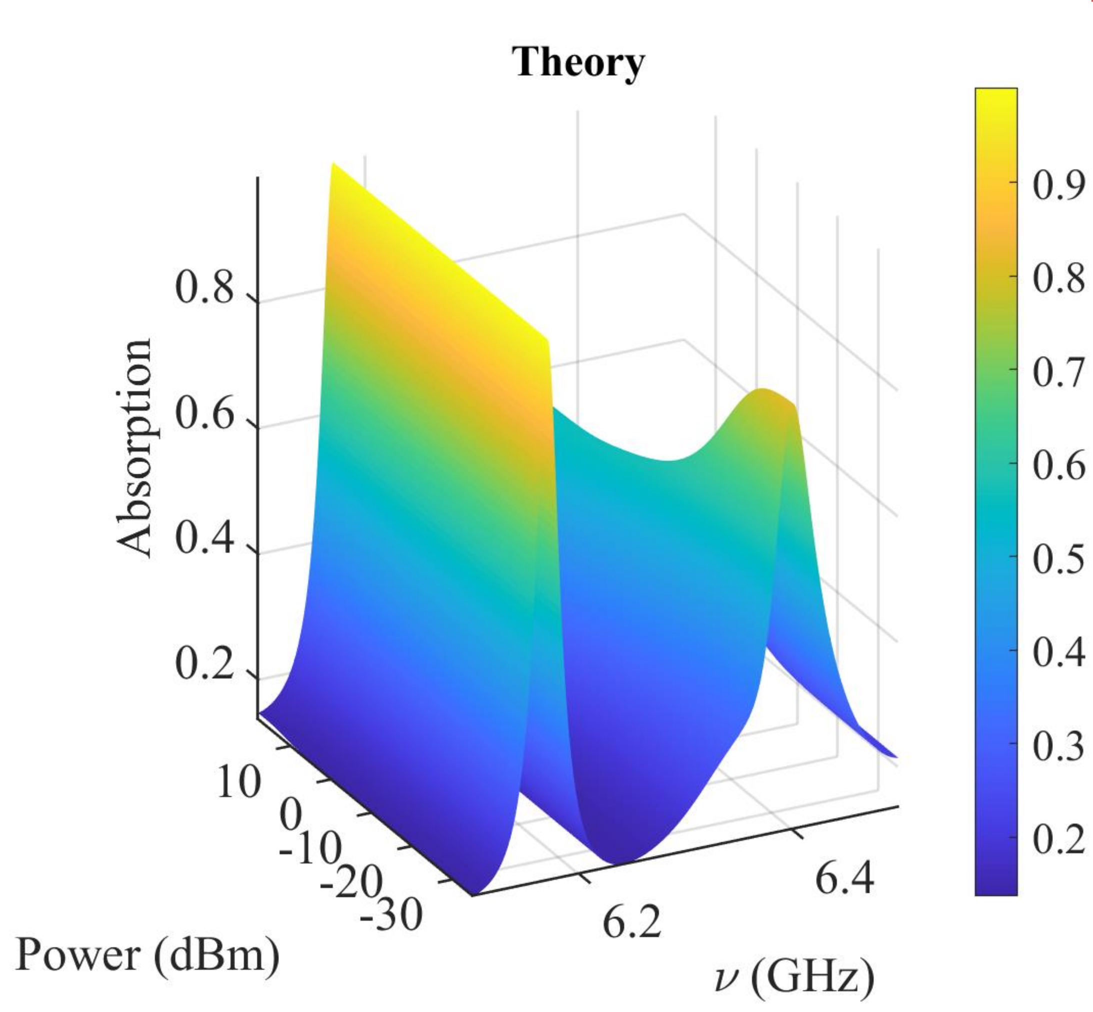


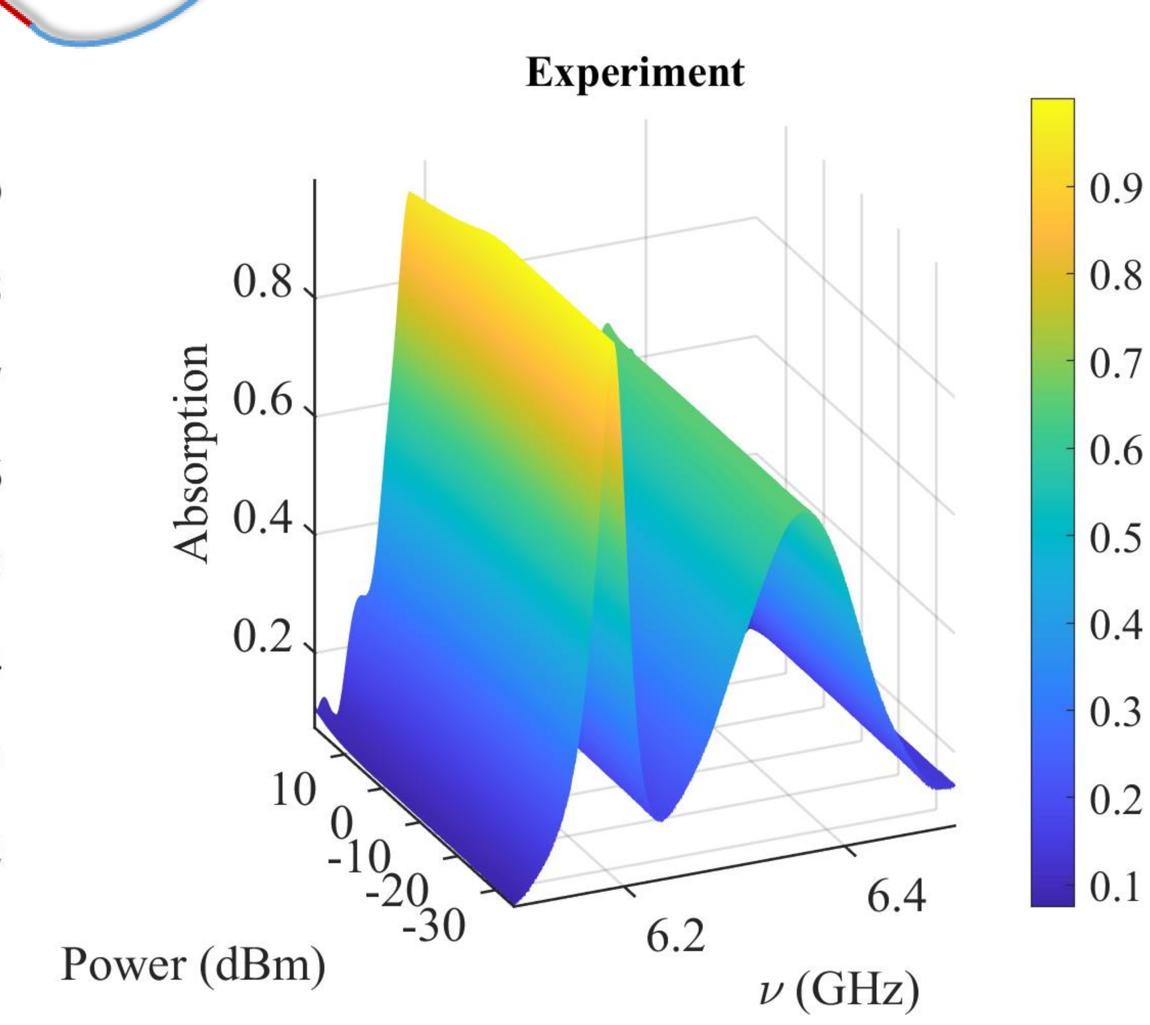




$$\tilde{\omega} = \omega_0 + Z_0 - \frac{Z_1}{1+\alpha|a(t)|^2}$$







2. Numerical Procedure

Re-write continuity & current conservation, together with L bc, in matrix form

$$M(|\phi_N|^2;k)\Phi = 0; \qquad \Phi = (\phi_1,\phi_2,\cdots,\phi_N)^T$$

$$M_{\mu,\beta} = \begin{cases} -\sum_{\gamma\neq\mu} A_{\mu\gamma}\cot kL_{\mu\gamma} + \lambda_\mu k + \delta_{\mu,N}\chi k|\Phi_N|^2 - i\delta_{\mu,1} - i\delta_{\mu,2}, & \mu = \beta \\ A_{\mu\beta}\csc kL_{\mu\beta}, & \mu \neq \beta \end{cases}$$

$[M_{N-1}(k)]$ is $(N-1)\times(N-1)$ submatrix of $M(|\phi_N|^2;k)$

Separate the equations involving the linear nodes from the equation involving the nonlinear node

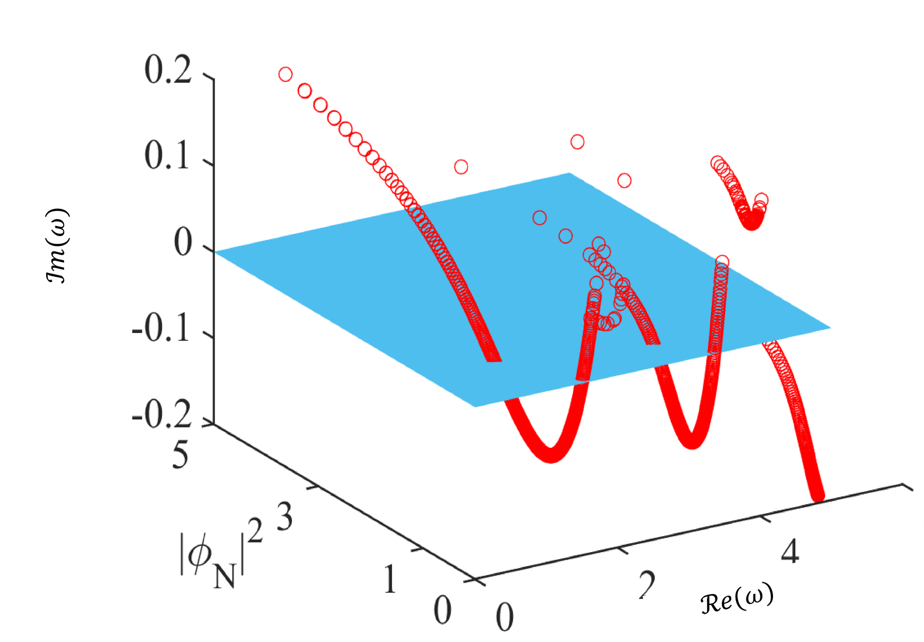
$$M_{N,N}(|\phi_N|^2;k) = (M_{N,1}(k),\cdots M_{N,N-1}(k))[M_{N-1}(k)]^{-1}\begin{pmatrix} M_{1N}(k) \\ M_{2N}(k) \\ \vdots \\ M_{N-1,N}(k)\end{pmatrix}$$

Constrains:

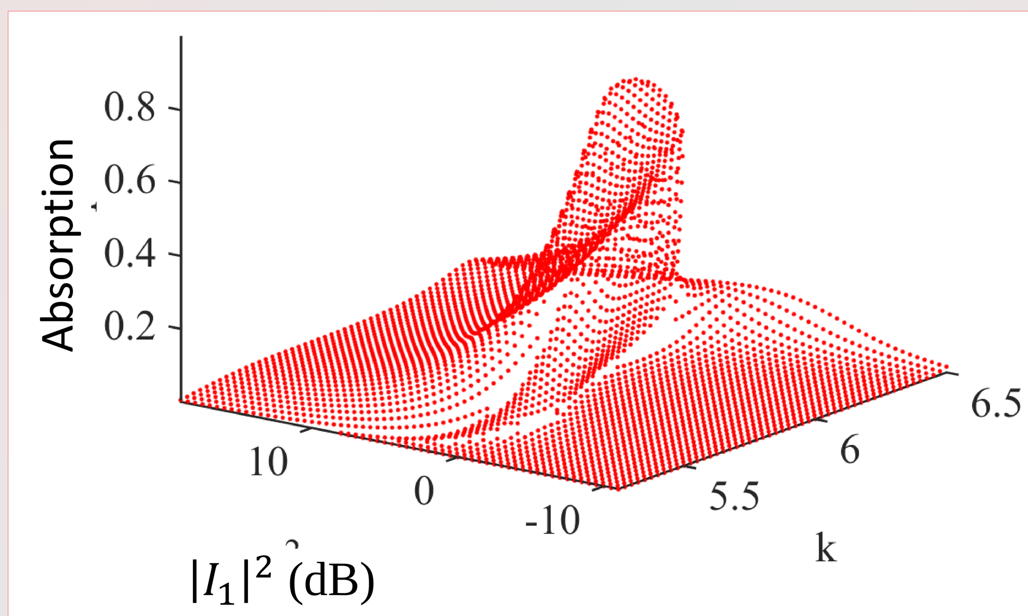
$$0 \le |\phi_N|^2 \in R, \qquad k \in R$$

NLCPA fields are evaluated from continuity:

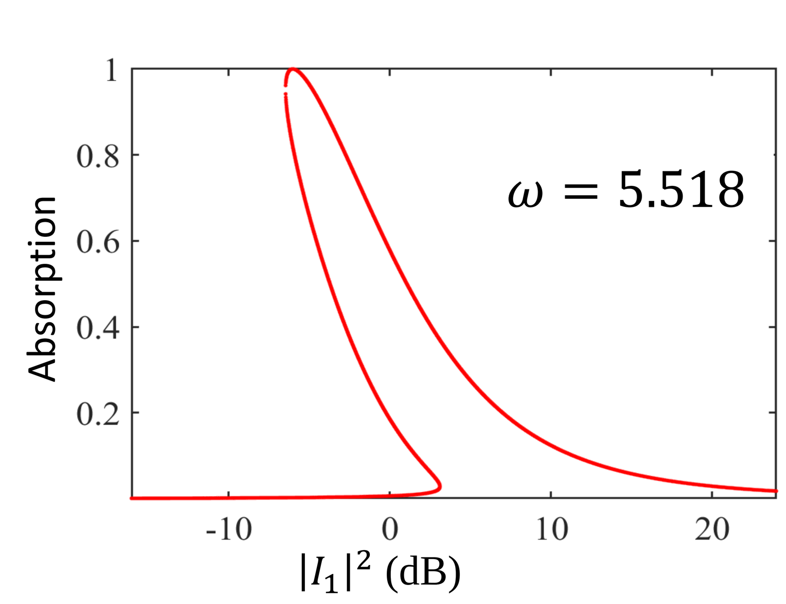
$$\phi_1 = I_1; \quad \phi_2 = I_2$$



4. Absorbance for NLCPA wavefronts



$$A(\omega) = 1 - \frac{(|O_1|^2+|O_2|^2)}{(|I_1|^2+|I_2|^2)}$$



References

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[2] H. Li, S. Suwunnarat, R. Fleischmann, H. Schanz, and T. Kottos, Phys. Rev. Lett. 118, 044101 (2017)
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