Statistical Description of Electromagnetic Wavefronts for Extreme Near-field Micromanipulations

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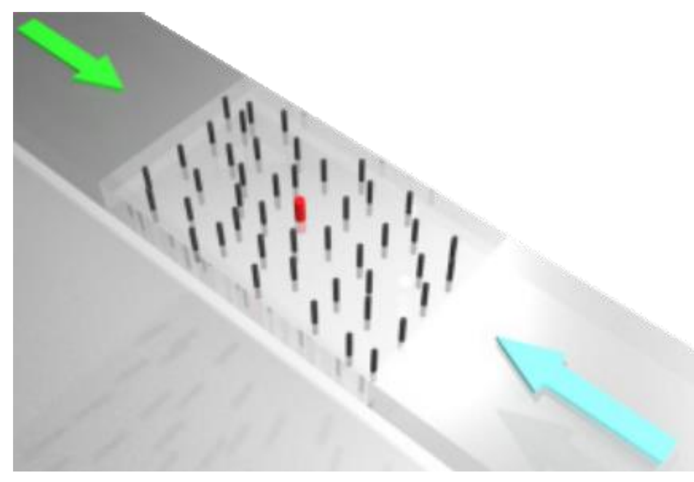
Abstract

We design a wavefront shaping protocol which maximizes the electromagnetic forces acting at a specific location in an arbitrary complex scattering medium. Our approach utilizes appropriate physical operators, that rely on the measured scattering matrix, whose eigenvectors can be used for the design of wavefronts in the far field with optimal properties (e.g. extreme force, pressure, etc.) in the near field of a target. A statistical description of these optimal waveforms is performed using a universal coupled mode theory formalism whose results are tested against an actual complex system consisting of a complex network of microwave graphs with an embedded localized target.

1. Coupled Mode Theory

$$\omega|\psi\rangle = \hat{H}_{eff}|\psi\rangle + i\hat{D}^T|s^+\rangle$$
$$|s^-\rangle = \hat{D}|\psi\rangle - |s^+\rangle$$
Green's function: $\hat{G} = \left(\omega - \hat{H}_{eff}\right)^{-1}$
$$|\psi\rangle = i\hat{G}\hat{D}^T|s^+\rangle$$
Scattering Matrix: $\hat{S} = -\hat{1}_M + i\hat{D}\hat{G}\hat{D}^T$

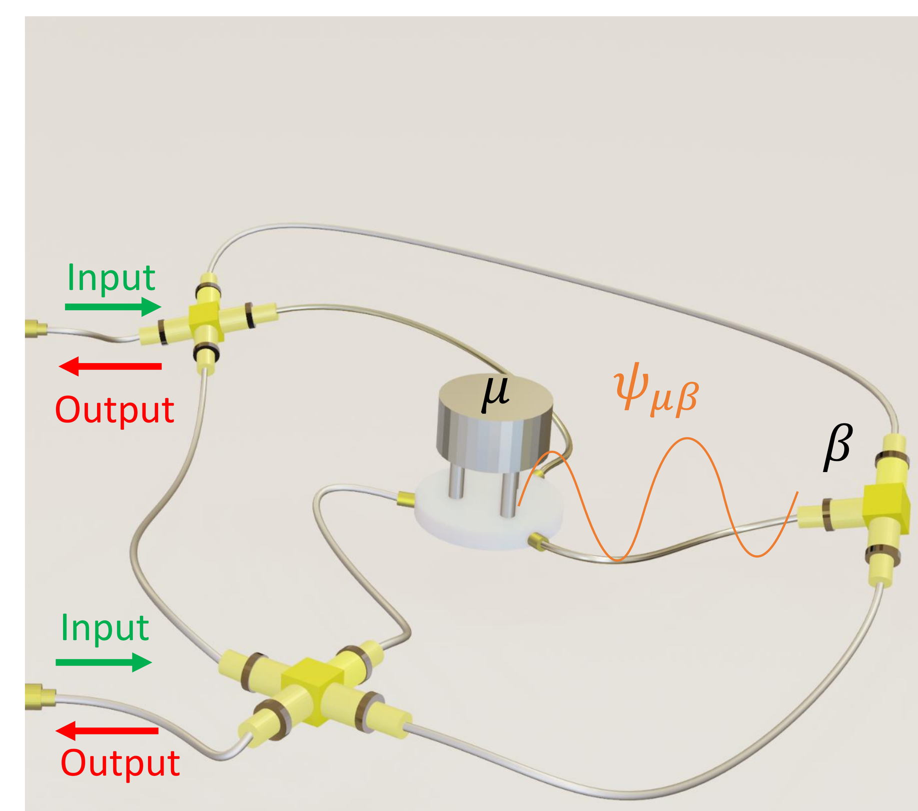


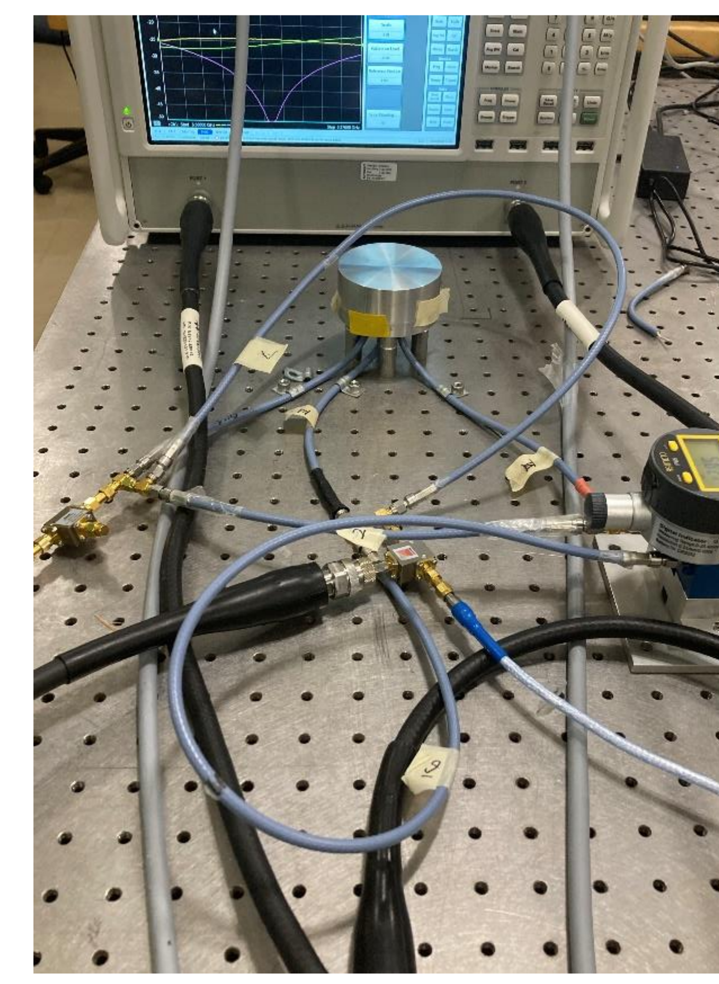
2. Linear Graphs

Wave Equation:
$$\frac{d^2}{dx^2}\psi_{\mu\beta}(x) + \frac{\omega^2\varepsilon_0}{c^2}\left(1+\lambda_\mu\delta(x)\right)\psi_{\mu\beta}(x) = 0$$

Boundary Conditions at Vertices:
1. Continuity of Potential Difference
2. Current Conservation

Scattering Matrix:
$$\hat{S} = -\hat{1}_M + 2i\hat{W}\hat{R}\hat{W}^T$$





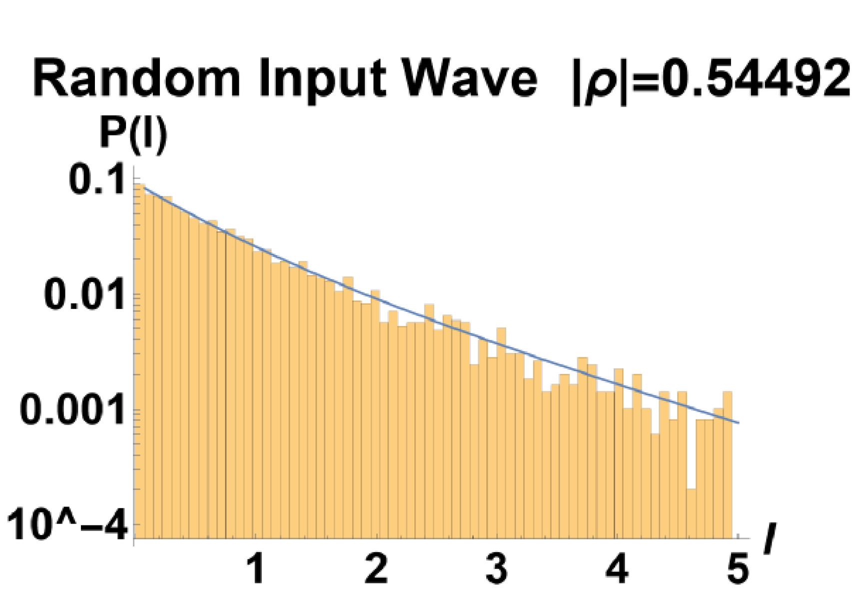
3. GWS Operators in CMT and Graphs

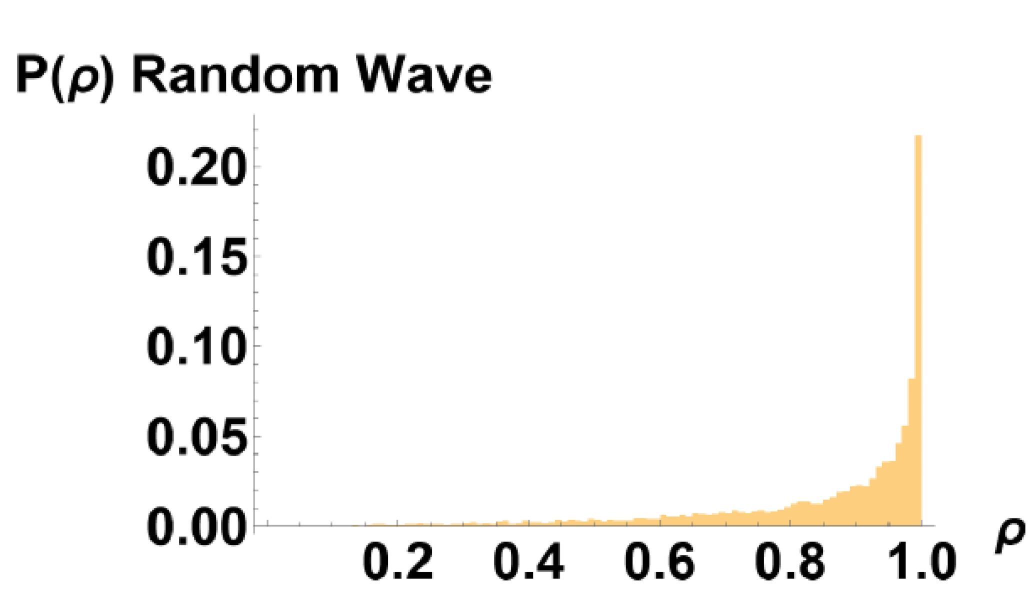
Operator Definition:
$$\hat{Q}_x = -i\hat{S}^\dagger\frac{d\hat{S}}{dx}$$
$$\langle\hat{Q}_x\rangle = \langle\boldsymbol{\mathcal{F}}\rangle \quad \text{where } \boldsymbol{\mathcal{F}} = -\frac{d}{dx}\hat{H}_{eff}$$

Model Framework	CMT	Graphs
GWS for Unitary Systems	$\hat{Q}_x = -\hat{D}\hat{G}^\dagger\frac{d}{dx}(\hat{H}_{eff})\hat{G}\hat{D}^T$	$\hat{Q}_x = 2k\hat{W}\hat{R}^\dagger\frac{d}{dx}(\hat{R}^{-1})\hat{R}\hat{W}^T$
Expectation Value (Generalized Force)	$\langle s^+\|\hat{Q}_x\|s^+\rangle = -\left\langle\psi\left\|\frac{d}{dx}\hat{H}_{eff}\right\|\psi\right\rangle$	$\langle I\|\hat{Q}_x\|I\rangle = 2k\left\langle\varphi\left\|\frac{d}{dx}\hat{R}^{-1}\right\|\psi\right\rangle$
GWS for target perturbation (n,μ)	$\langle\hat{Q}_{\varepsilon_n}\rangle = -\langle\psi\|\hat{P}_n\|\psi\rangle = -\|\psi_n\|^2$ With $[\hat{P}_n]_{ij} = \delta_{i,n}\delta_{j,n}$	$\langle\hat{Q}_{\lambda_\mu}\rangle = 2k\langle\varphi\|\hat{P}_\mu\|\varphi\rangle = 2k\|\phi_\mu\|^2$ With $[\hat{P}_\mu]_{ij} = \delta_{i,\mu}\delta_{j,\mu}$

4. GWS Statistics in CMT

Site intensities resulting from random input wave are given by,
$$P_\rho(I) = \frac{1}{\sqrt{1-|\rho|^2}}\exp\left[-\frac{I}{1-|\rho|^2}\right]\mathrm{I}_0\left[\frac{|\rho|I}{1-|\rho|^2}\right]$$
With $\rho = \frac{\psi\cdot\psi}{|\psi|^2}$, $|\rho|^2$ is the phase rigidity

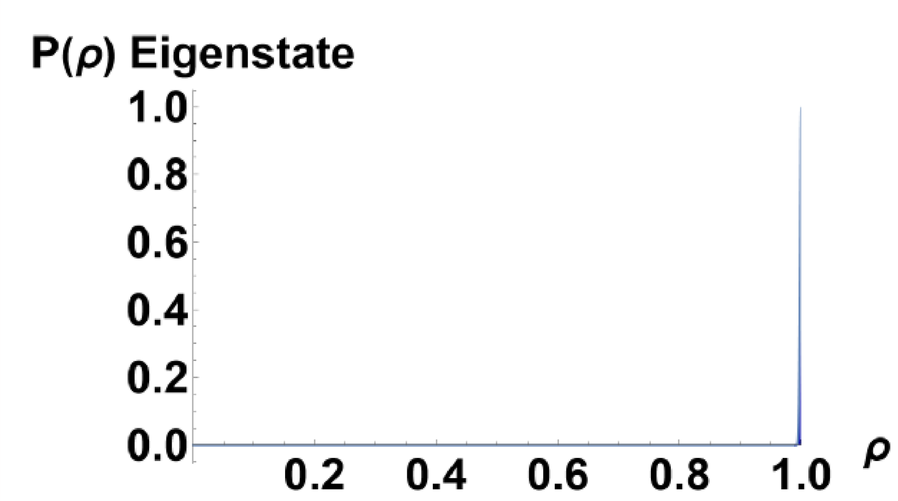


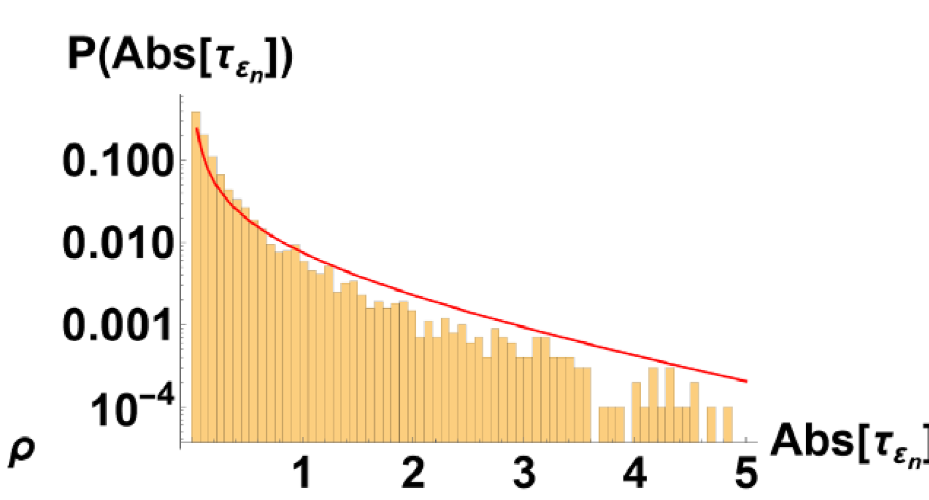


5. GWS Maximal Eigenvalue Statistics

As $|\rho|\to 1$, $P_\rho(I)$ becomes the Porter-Thomas Distribution
$$P(I) = \frac{1}{\sqrt{2\pi I}}\exp\left[-\frac{I}{2}\right]$$
$|\rho|\approx 1$ corresponding to maximal eigenstate of $\hat{Q}_{\varepsilon_n}$

Maximal Eigenvalue of $\hat{Q}_{\varepsilon_n}$:
$$\tau_{\varepsilon_n} = -2\gamma_e\sum_{r=1}^{M}\left|[\hat{G}]_{n,r}\right|^2$$
with assoc. eigenstate $\langle l|s^+\rangle = [\hat{G}]^*_{n,l}$





Acknowledgements

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